FusyBots Robotics Formula Sheet

Configuration Space, Rotation Frames & Rigid Body Motion

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1. Configuration Space (C-Space)

Robot Configuration

Formula: q = (q1, q2, ..., qn)

Why it matters: Represents the complete state of a robot using joint variables.

Degrees of Freedom (DOF)

Formula: DOF = m(N – 1 – J) + Σfi

Why it matters: Defines independent variables required for robot motion.

Configuration Space

Formula: C = {q}

Why it matters: Represents all possible robot configurations.

Collision-Free Space

Formula: C_free = C – C_obs

Why it matters: Used in motion planning and obstacle avoidance.

Holonomic Constraints

Formula: g(theta) = 0

Why it matters: Position-based constraints used in closed-chain robots.

Nonholonomic Constraints

Formula: A(theta) * theta_dot = 0

Why it matters: Velocity constraints used in wheeled robots.

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2. Rotation Frames

Rotation Matrix

Formula: $R \in SO(3)$

Why it matters: Represents orientation between coordinate frames.

Rotation Matrix Properties

Formula: $R^T R = I$, $\det(R)=1$

Why it matters: Ensures valid rigid body rotations.

2D Rotation Matrix

Formula: $R(\theta) = [[\cos\theta, -\sin\theta], [\sin\theta, \cos\theta]]$

Why it matters: Used in planar robot transformations.

Angular Velocity (Space Frame)

Formula: $[\omega_s] = \dot{R} * R^{-1}$

Why it matters: Angular velocity relative to world frame.

Angular Velocity (Body Frame)

Formula: $[\omega_b] = R^{-1} * \dot{R}$

Why it matters: Angular velocity relative to body frame.

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3. Rigid Body Motion

Homogeneous Transformation Matrix

Formula: $T = [[R, p], [0, 1]]$

Why it matters: Combines rotation and translation into one representation.

Transformation Inverse

Formula: $T^{-1} = [[R^T, -R^T p], [0,1]]$

Why it matters: Used for frame conversion and pose transformations.

Point Transformation

Formula: $p_s = R * p_b + t$

Why it matters: Transforms a point between coordinate frames.

Twist Representation

Formula: V = [omega, v]

Why it matters: Represents rigid body linear and angular velocity.

Screw Motion

Formula: T = exp([S] theta)

Why it matters: Used in advanced robot motion representation.

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4. Real Robotics Connections

Concept	Used In
C-Space	Motion Planning
Rotation Matrix	Frame Transformations
Transformation Matrix	ROS TF
Twist	Mobile Robots
Screw Motion	Manipulator Motion
Angular Velocity	Drone Stabilization

Most Important Intuition:
Robots think using transformations, coordinate frames, and configuration spaces.

Built with ❤■ by FusyBots